MAI 2 3. a 4. cvičení - neurčitý integrál 3.

(Najděte primitivní funkce na maximálních otevřených intervalech.)

Vhodné substituce, vedoucí na integraci racionální funkce.

1. $\int R(e^x)\,dx$ - substituce $e^x = t$:

$$\int \frac{1}{e^{2x}+e^x-2}\,dx\ ;\ \int \frac{1}{e^{2x}+2e^x+2}\,dx\ ;\ \int \frac{2e^{2x}-5}{e^{2x}+4e^x+5}\,dx\ ;\ \int \frac{e^x-1}{e^x+1}\,dx\ ;\ \int \frac{1}{(e^x+1)^2}\,dx\ ;$$

2. $\int R(\log x)\frac{1}{x}dx$ - substituce $\log x = t$:

$$\int \frac{\log x}{x(\log x-1)(\log^2 x-2\log x+2)}\,dx\ ;\ (*)\ \int \frac{\log x+1}{x(\log^3 x+8)}\,dx\ ;$$

3. $\int R\left(x,\sqrt[s]{\frac{ax+b}{cx+d}}\right)dx$, $s\in N, ad-bc\neq 0$ - substituce $\sqrt[s]{\frac{ax+b}{cx+d}}=t$:

$$\int \frac{1}{1+\sqrt{x}}\,dx\ ;\ \int \frac{\sqrt{x}-1}{x\cdot(x-2\sqrt{x}+2)}\,dx\ ;\ \int \frac{1}{(\sqrt{x}+2)\cdot(x+6\sqrt{x}+10)}\,dx\ ;\ \int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}\,dx\ ;$$

$$\int \frac{1+\sqrt[4]{x}}{x+\sqrt{x}}\,dx\ ;\ \int \frac{1}{x^2}\cdot\sqrt{\frac{1+x}{x}}\,dx\ ;\ \int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\,dx\ ;\ \int \frac{1}{\sqrt{x+1}+\sqrt[3]{x+1}}\,dx\ ;$$

4. $\int R(x,\sqrt{ax^2+bx+c})\,dx$ - vhodné substituce:

$a>0$: $\sqrt{ax^2+bx+c}=\pm\sqrt{a}\,x\pm t$ nebo $c>0$: $\sqrt{ax^2+bx+c}=\sqrt{c}+xt$ (Eulerovy substituce)

$a<0$ a polynom ax^2+bx+c má dva různé reálné kořeny $\alpha_1<\alpha_2$: pak lze

$$\sqrt{ax^2+bx+c}=\sqrt{-a}\,(x-\alpha_1)\sqrt{\frac{\alpha_2-x}{x-\alpha_1}} \text{ a substituovat } \sqrt{\frac{\alpha_2-x}{x-\alpha_1}}=t \text{ nebo}$$

$$\sqrt{ax^2+bx+c}=\sqrt{-a}\,(\alpha_2-x)\sqrt{\frac{x-\alpha_1}{\alpha_2-x}} \text{ a substituovat } \sqrt{\frac{x-\alpha_1}{\alpha_2-x}}=t$$

(a jiné vhodné substituce):

(i) $\int \frac{1}{\sqrt{x^2+1}}\,dx$; $\int\sqrt{x^2+1}\,dx$ (také $x=\sinh t$ ($\sinh t=\frac{e^t-e^{-t}}{2}$));

$\int \frac{1}{x\sqrt{x^2+1}}\,dx$ (zkuste také $t=\frac{1}{x}$, $t=1+x^2$, $t=\sqrt{1+x^2}$ nebo $x=\sinh t$);

$\int \frac{1}{\sqrt{x^2-1}}\,dx$ (také $x=\cosh t$ ($\cosh t=\frac{e^t+e^{-t}}{2}$) a platí $\cosh^2 t-\sinh^2 t=1$);

$\int \frac{1}{x\sqrt{x^2-1}}\,dx$; $\int \frac{\sqrt{1+x^2}}{x}\,dx$; $\int \frac{1}{\sqrt{x^2+x+1}}\,dx$; $\int \frac{1}{x\sqrt{x^2+x+1}}\,dx$ (zkuste také $t=\frac{1}{x}$);
$\int \frac{1}{\sqrt{(x^2+x+1)^3}}\,dx$; $\int \frac{1}{\sqrt{x^2+2x+3}}\,dx$; $\int \frac{1}{x+\sqrt{x^2+x+1}}\,dx$; $\int \frac{1}{1+\sqrt{x^2+x+2}}\,dx$;

(ii) $\int \frac{1}{\sqrt{2+x-x^2}}\,dx$; $\int \frac{1}{x\sqrt{2+x-x^2}}\,dx$; $\int \frac{1}{x+\sqrt{2+x-x^2}}\,dx$; $\int \frac{x}{\sqrt{6+x-x^2}}\,dx$; $\int \frac{x^2}{\sqrt{9-x^2}}\,dx$.

5. $\int R(\sin x,\cos x)\,dx$:

(i) substituce $\sin x = t$ nebo $\cos x = t$:

$\int \frac{\cos x}{\sin^3 x}\,dx$; $\int \frac{\sin x}{2+\cos x}\,dx$; $\int \frac{\sin x}{(1-\cos x)^2}\,dx$; $\int \sin^5 x\,dx$; $\int \sin^3 x\cdot\cos^2 x\,dx$; $\int \frac{1}{\sin x}\,dx$;
$\int \frac{1}{\cos^3 x}\,dx$; $\int \frac{1}{(2+\cos x)\sin x}\,dx$;

(ii) substituce $tg\,x = t$:

$\int \frac{1}{1+tgx}\cdot\frac{1}{\cos^2 x}\,dx$; $\int \frac{1}{1+tgx}\,dx$; $\int \frac{\sin^2 x}{1+\sin^2 x}\,dx$; $\int \frac{1}{\cos^4 x}\,dx$; $\int \frac{1}{\sin^2 x\cdot\cos^2 x}\,dx$;
$\int \frac{1}{(\sin x+\cos x)^2}\,dx$;

(iii) substituce $tg\,\frac{x}{2} = t$:

$\int \frac{1}{2+\cos x}\,dx$; $\int \frac{1}{5+4\sin x}\,dx$; $\int \frac{2+\sin x}{2-\sin x}\,dx$; $\int \frac{2-\sin x}{2+\cos x}\,dx$.